Dc-free codes of rate $(n-1)/n$, n odd

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Summary – *We report on a new class of dc-free codes of rate $(n-1)/n$, n is odd. Spectral and runlength properties of the new codes have been evaluated by computer simulation.*

Key words: dc-free code, constrained code, magnetic recording

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1 Introduction

Binary sequences with spectral nulls at zero frequency, or also called *matched spectral null codes* have found widespread application in communication and recording systems. Let $\{x_0, \ldots, x_i, \ldots\}$, $x_i \in \{-1, 1\}$ be a bipolar sequence. The (running) digital sum z_i is defined as

$$z_i = \sum_{j=-\infty}^{i} x_j = z_{i-1} + x_i.$$

If z_i is bounded, the spectral density of the sequence $\{x_i\}$ vanishes at zero frequency [1]. Then, at any instant i the RDS z_i assumes a finite number of sum values, N, called *digital sum variation* (DSV). Traditionally, binary input symbols $y_i \in \{0, 1\}$ are translated into the bipolar channel bits x_i using a precoder and a range converter. The precoder's operation can be written as

$$\hat{x}_i = \hat{x}_{i-1} \oplus y_i, \quad x_i = 2\hat{x}_i - 1, \quad 1 \le i \le n, \tag{1}$$

where $\oplus$ denotes GF(2) addition. The various signals are shown in Figure 1.

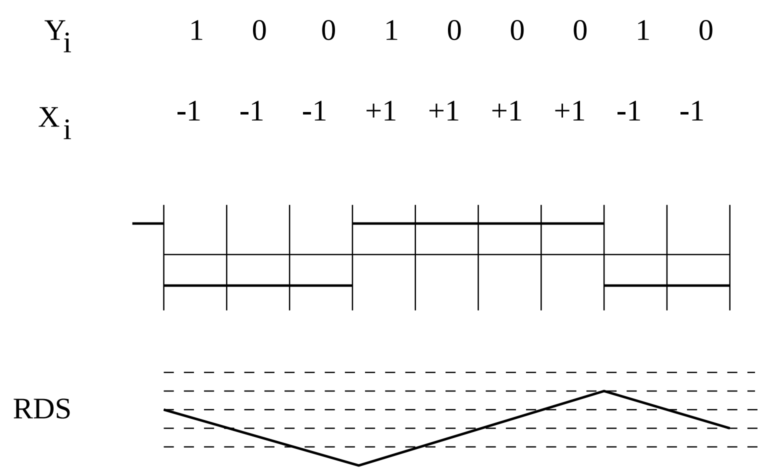

Figure 1: Running digital sum (RDS) versus time. The binary input symbols y_i are translated into the bipolar channel bits x_i using a precoder and a range converter. The other curves show the relationship between the sequence $\{x_i\}$, the write signal and the running digital sum (RDS).

In this Correspondence, we will focus on state-dependent codes of rate m/n. In its simplest form, the set of encoder states, called *principal states*, is a subset of the N channel states $\{z_i\}$. From each of the principal states there are at least 2^m constrained words beginning at such a state and ending in a principal state. The set of principal states can be found by invoking Franaszek's procedure [2]. The structure of the codes at hand allows a simpler approach.

Let $\boldsymbol{x}$ denote the n-tuple $(x_1, \ldots, x_n)$, where $x_i \in \{-1, 1\}$, then we define the disparity of $\boldsymbol{x}$ as

$$d(\boldsymbol{x}) = \sum_{i=1}^{n} x_i.$$

If we require that the running digital sum after concatenation of a new codeword is not larger (in absolute terms) than that at the beginning then each source word must have representation of zero-disparity or it must have two alternative representations of opposite disparity. The words available can easily be computed. Let N_- and N_+ denote the number of codewords with $d(\boldsymbol{x}) \leq 0$ and $d(\boldsymbol{x}) \geq 0$, respectively. Then, we find

$$N_- = N_+ = \begin{cases} 2^{n-1}, & n \text{ odd}, \\ 2^{n-1} + \frac{1}{2}\begin{pmatrix} n \\ \frac{n}{2} \end{pmatrix}, & n \text{ even}. \end{cases} \qquad (2)$$

When n is even, we are in the comfortable position that we can choose 2^{n-1} codewords from the many candidates available. For n odd, on the other hand, it can be seen that there are just enough codewords available to cater for a rate $(n-1)/n$ code. The implementation of the latter code is usually termed *polarity switch* code, where $(n-1)$ source symbols are supplemented by one symbol called the *polarity bit* [3] [1]. The encoder has the option to transmit the n-bit words "as is" or to invert all symbols. The choice of a specific translation is made in such a way that the RDS after transmission of the word is as close to zero as possible. The polarity bit is used by the decoder to undo the action of the encoder. Spectral properties of the polarity bit code have been investigated by Greenstein [4] and Immink [1]. From the above deliberation, it seems, at first glance, that, for n odd, no codes other than the polarity switch method are feasible. In the next section, we will demonstrate otherwise.

2 The new construction

The crux of the new class of dc-free code is the redefinition of the running digital sum. In the classical definition, see Figure 1, it is assumed that transitions occur at the start of the bit cells. In the new definition, transitions are assumed to occur halfway the bit cells. Figure 2 shows the various signals. Note that the new definition does not have a physical meaning. Essentially the write signals are delayed by half a channel bit interval. Let x_0 be the value of the bit preceding the word $\boldsymbol{x}$. Then the disparity, $ds(\boldsymbol{y}, x_0)$, of the binary n-tuple $\boldsymbol{y}$ after precoding using the new definition is

$$ds(\boldsymbol{y}, x_0) = \sum_{i=1}^{n} x_i + (w(\boldsymbol{y}) \bmod 2)x_0, \qquad (3)$$

where $w(\boldsymbol{y})$ denotes the weight of $\boldsymbol{y}$. The first right-hand term is the "conventional" contribution to the RDS, and the second term expresses a correction term. The interesting result of this new definition is that zero-disparity i.e. $ds(\boldsymbol{y}, x_0) = 0$, codewords, are possible for n odd.

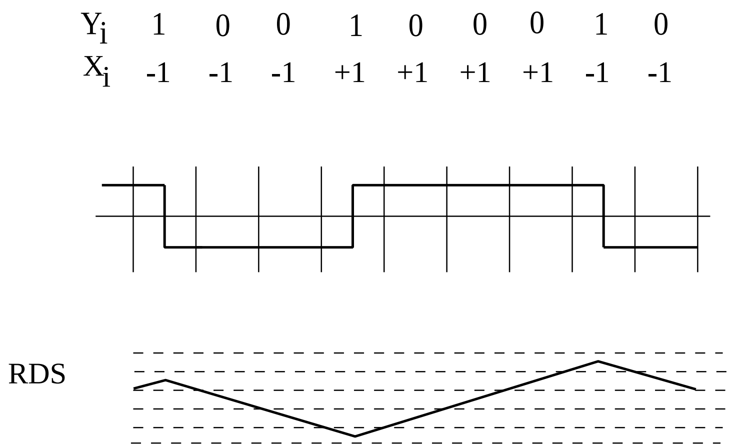

Figure 2: Running digital sum (RDS) versus time. The binary input symbols y_i are translated into the bipolar channel bits x_i using a precoder and a range converter. The transitions occur halfway the bit cells. Note that the contribution to the RDS of the 9-bit word $\boldsymbol{y}$ is nil, while, see Figure 1, using the conventional definition the contribution of the same input word $\boldsymbol{y}$ is -1.

Let $N(s)$ denote the number of words $\boldsymbol{y}$ having $ds(\boldsymbol{y}, 1) = s$. Then, it is not difficult to see that $N(s)$ can be found with

$$N(s) = \binom{n-1}{\frac{n-1}{2} + \lfloor \frac{s}{2} \rfloor}. \qquad (4)$$

The zero-disparity words are uniquely allocated to the source words. The other codewords are allocated in pairs of opposite disparity. The The choice of a specific representation is made to minimize the absolute value of the running digital sum. The words available in both modes can easily be computed. To that end, let $\mathcal{N}_-$ and $\mathcal{N}_+$ denote the sets whose members $\boldsymbol{y}$ satisfy $ds(\boldsymbol{y}, 1) \leq 0$ and $ds(\boldsymbol{y}, 1) \geq 0$, respectively, and let N_- and N_+ denote the cardinality of these sets. Then,

$$N_- = 2^{n-1} \qquad (5)$$

and

$$N_+ = 2^{n-1} + \begin{pmatrix} n-1 \\ \frac{n-1}{2} \end{pmatrix}. \tag{6}$$

From Eqs. (5) and (6) we infer that, as $N_- = 2^{n-1}$, the designer has no other choice than taking the words available, whereas, as $N_+ > 2^{n-1}$, the designer has the freedom to choose words that satisfy certain design criteria such as low coder/decoder complexity, minimizing maximum runlength, etc. It can easily be verified that both $10^{n-1} \in \mathcal{N}_-$ and $0^{(n-1)/2}10^{(n-1)/2} \in \mathcal{N}_-$, where 0^p denotes a string of p zeros. We conclude therefore that the maximum 'zero'-runlength equals $3(n-3)/2$.

3 Spectral Performance

The spectral performance of the new codes has been investigated by computer simulation. As an example we studied codes of rate 8/9. The number of codewords with non-negative disparity, N_+, equals 326. We require 256 words, and from the codewords available we chose the words with the smallest absolute disparity. The maximum runlength is twelve. The rate 8/9, polarity-switch code shows a 4 dB less rejection at the low-frequency end. Also the maximum 'zero'-runlength is much larger, 18, than that of the new code. The quantity *sum variance*, which is inversely proportional to the width of the spectral notch at zero frequency, has been adopted to quantify the low-frequency properties of a dc-free code [1]. Table 1 lists the sum variance of the new code, s^2, and the sum variance, s_P^2, of the polarity switch code versus codeword length n. It can be seen that the new codes perform better as they show a smaller sum variance than that of the conventional scheme.

Table 1: Sum variance of the new code, s^2, and the sum variance, s_P^2, of the polarity switch code versus codeword length n.

n	s^2	s_P^2
3	1.05	1.67
5	1.73	3.00
7	2.52	4.33
9	3.23	5.66
11	4.07	7.00

4 Conclusions

We have reported on a new class of rate $(n-1)/n$, n odd, dc-free codes which have been designed on the basis of a redefinition of the running digital sum. Properties of the new codes, such as spectral and runlength distribution have been evaluated by computer simulation.

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Figure Captions

Figure 1 - Running digital sum (RDS) versus time. The binary input symbols y_i are translated into the bipolar channel bits x_i using a precoder and a range converter. The other curves show the relationship between the sequence $\{x_i\}$, the write signal and the running digital sum (RDS).

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Table Caption

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