A New DC-free Runlength Limited Coding Method for Data Transmission and Recording

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***Abstract*—This paper describes a new coding method based on binary (d, k) runlength constraints used for recording or transmitting an audio or video signal, computer data, etc. Data words of m bits are translated into codewords of n bits using a conversion table. The codewords satisfy a (d, k) runlength constraint in which at least d and not more than k '0's occur between consecutive '1's. The n-bit codewords alternate with p-bit merging words which in the prior art are selected such that the d and k are satisfied at the borders of consecutive codewords. We present a new coding method, where the codewords obey the (d, k)-constraint, but the merging words are not required to obey the (d)-constraint. The merging word that satisfies said conditions, yielding the lowest low-frequency spectral content of the encoded signal obtained after modulo-2 integration, is selected. The spectral performance of the new coding method has been appraised by computer simulations for the EFM (Eight-to-Fourteen Modulation) parameters, $d = 2, k = 10$, and $p = 3$. The low-frequency content of the signal generated by the newly presented coding method is around 4 dB lower in the relevant low-frequency range than that generated by the conventional EFM method.**

***Index Terms*—Constrained coding, runlength limited code, Eight-to-Fourteen Modulation, EFM.**

I. Introduction

Runlength limited (RLL) codes have been widely applied in well-known consumer electronics products such as magnetic recording products and optical discs including Compact Disc, DVD, and Blu-Ray Disc [1]. RLL codes have recently been advocated in visible light communications (VLC) systems, where light intensity of solid-state light sources, mostly LEDs, are varied [2]. Experimental DNA-based storage architecture using RLL codes was implemented by Church and others [3, 4, 5, 6].

Binary RLL sequences are characterized by two parameters, $d + 1$ and $k + 1$, which stipulate the minimum and maximum runlength, respectively, where a runlength is the number of consecutive like symbols in the sequence. In RLL sequences, the occurrence of very short and/or long runlengths are avoided. If the runlength is too short, intersymbol interference may become too excessive, resulting in read or transmission errors. The maximum runlength constraint ensures that adequate timing information can be derived from the received signal. An RLL sequence is usually generated by a transformation of a (d, k)-constrained sequence, which has at least d and at most k zero's between consecutive one's. Modulo-2 integration of a (d, k)-constrained sequences delivers an RLL sequence with runlengths between $d + 1$ and $k + 1$.

Block codes have been used to translate user data into a (d, k)-constrained sequence. In a block code format, the user data is partitioned into blocks of m bits, which are uniquely translated into n bits that conform the prescribed d and k runlength constraints. The n-bit blocks are cascaded to form a long sequence of symbols, and subsequently forwarded to the recorder or transmitter. In order to satisfy the (d, k)-constraint between the cascaded n-bit blocks, p-bit *merging blocks* are inserted between consecutive n-bit blocks. A seminal construction method presented by Tang and Bahl [7], has $p = d+2$. Improved construction methods which impose additional constraints on the codewords allow $p = d$ [1].

In addition, the low-frequency components of the encoded signal should be kept as small as possible. Such a signal is called a *dc-free* signal. A first reason for using said dc-free signals is that recording or transmission channels are not normally responsive to low-frequency components. The suppression of low-frequency components in the signal is also highly advantageous, for example, when the signal is read from an optical disc on which the signal is recorded in the track, because then continuous tracking control undisturbed by the recorded signal is possible. A good suppression of the low-frequency components leads to improved tracking with less disturbing audible noise. For visible light communication it is desirable that the intensity variation of the light is invisible to the users, that is, annoying *flicker* should be mitigated [8]. This requirement implies that the spectrum of the encoded signal should not contain low-frequency components.

Eight-to-Fourteen-Modulation (EFM), a dc-free block code with basic parameters $(d = 2, k = 10)$, and $p = 3$, has been used in the Compact Disc [1]. It is compulsory that the p(=3)-bit merging words satisfy the $(d = 2, k = 10)$ runlength constraints imposed for the cascaded sequence. They are further selected to minimize the low-frequency content of the encoded sequence.

An improved method for suppressing the low-frequency components of RLL block codes is described in [9]. In said article, the authors describe a method where the selection of a p-bit merging word does not only depend on a single upcoming codeword, but where in contrast the selection is made using q, $q > 1$ upcoming codewords. From the article cited, it can be concluded that this look-ahead strategy

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improves the quality of the low-frequency suppression. A significant disadvantage of said strategy is the difficulty of implementing it as the number of operations, such as adding, comparing, buffering, and so on, grows exponentially with the number of codewords involved in the selection process.

In [10], Immink describes a type of constrained codes, termed *weakly constrained* codes. Weakly constrained codes fail to comply with the (d, k)-constraints as they produce sequences that violate the prescribed constraints with (low) probability. It is argued that if the channel is not free of errors, it is pointless to feed the channel with perfectly constrained sequences. Violation of prescribed (d, k)-constraints offers an additional degree of freedom that can be exploited to reduce the low-frequency components. It is known that violations of the (d)-constraint, i.e., runs of '0's less than d, are prone to error resulting from inter symbol interference. A violation of the (k)-constraint could easily result in loss of clock synchronization, which, in turn, could lead to a burst of errors. The method is therefore not very effective as the frequency of violations of the dk-constraint must be small as they will inevitably lead to bit detection errors at the receiver's site.

Information recording has a constant need for increasing the reading and writing speed. The aim of increased reading speed, however, requires higher servo bandwidth of the tracking mechanism, which, in turn, sets more severe restrictions on the suppression of the low-frequency components in the recorded signal. Improved suppression of the low-frequency components is also advantageous for suppressing audible noise arising from the tracking mechanism. For visible light communication systems, it is desirable that the intensity variation of the light, flicker, is invisible to the users. For this reason, it is desirable to make as many efforts to prevent the encoded signal from containing low-frequency components.

In this paper, we present a new construction method for designing weakly constrained (d, k) constrained block codes. The n-bit codewords obey the (d, k)-constraint, but the merging words are not required to obey the mandatory (d)-constraint. Violation of the prescribed d-constraint has a bearing on the error rate of the merging bits (and not the n-bit data). But as the p-bit merging words are skipped by the decoder, the (d)-constraint violations are harmless and as a result the data error rate is not affected. Violation of (d)-constraint of the p-bit merging words offers a greater degree of freedom for selecting the merging words, which has a bearing on the low-frequency content of the encoded data stream. The extra freedom for selecting the p-bit merging words, makes it possible to significantly reduce the low-frequency content of the encoded signal. We exemplify the new method by the design of an alternative and EFM-compatible coding method. The new method violates the regular EFM rules as the prescribed d(=2)-constraint is not obeyed in the 3-bit merging words. The spectral performance of the new coding method has been appraised by computer simulations, and we found that the new method performs around 4 dB better in the low-frequency range than the conventional EFM method.

We start in Section II with a description of the prior art. A description of the new method is given in Section III. Section IV concludes the paper.

II. Description of the Prior Art

Codes for optical disc recording are described in [11]. The (d, k) runlength constraint is imposed in all optical disc products as it is desirable that the system is self-clocking, which requires that consecutive transitions in the encoded signal should not be too far apart, and it is a further requirement that two transitions of the encoded signal should not be following too closely in order to limit inter symbol interference. In addition, the signal should be 'dc-free', that is the low-frequency components of the encoded signal should be kept as small as possible.

A. Eight-to-Fourteen-Modulation (EFM)

Eight-to-Fourteen-Modulation (EFM) developed by Immink and Ogawa in the early 1980s was adopted as the recording code for the Compact Disc [11]. The EFM signal is obtained by converting a series of m(=8)-bit information words into a series of n(=14)-bit codewords, and where p(=3) merging bits are inserted between consecutive codewords. Respective codewords of 14 bits satisfy the conditions that at least $d(= 2)$ and at most k(=10) '0's are placed between two consecutive '1's.

Under EFM rules, in order to satisfy this condition also between codewords, 3-bit merging words are used. It is easily verified that only four 3-bit merging words of the eight possible 3-bit merging words are permitted to be used, namely the words '001, '010', '000', and '100'. The remaining 3-bit merging words, namely '111', '011', '101', and '110' cannot used as they violate the prescribed d(=2)-constraint. One of the four allowed merging words is selected such that the bit string obtained after cascading alternate codewords and merging words firstly satisfies the (d, k)-constraint, and secondly that in the corresponding modulo-2 integrated signal the *running digital sum* (RDS) value remains substantially constant. The RDS at a specific instant is understood to mean the difference between the number of bit cells having the high signal value and the number of bit cells having the low signal value, calculated over the encoded signal portion situated before this specific instant. A substantially constant running digital sum value means that the frequency spectrum of the signal does not comprise frequency components in the low frequency range. There are instances where the merging word is not uniquely governed by the minimum and maximum runlength requirements. This freedom of choice is utilized for minimizing the power density at the low-frequency end. By deciding the merging words according to above rules, low-frequency components of the encoded signal can be reduced. Decoding of EFM signals is very simple. The 3-bit merging words

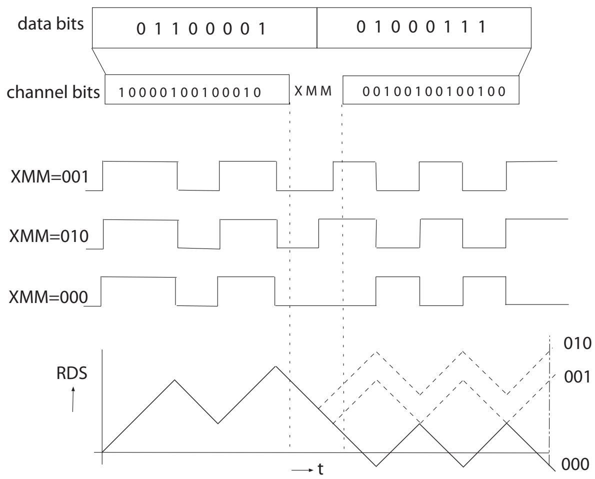

Fig. 1. Strategy for minimizing the running digital sum (RDS) in EFM. Two 8-bit user words are translated into 14-bit codewords using a look-up table. The 14-bit codewords, in turn, are cascaded, 'merged', using a 3-bit merging word in such a way that the runlength conditions continue to be satisfied for the cascaded sequence.

are skipped by the decoder, and the 14-bit codewords are translated, using a look-up table into the information bytes.

Figure 1 shows an example of the merging process. Eight user bits are translated into 14 channel bits using a look-up table. The 14-bit codewords are cascaded, 'merged', by means of 3-bit merging words in such a way that the runlength conditions continue to be satisfied. For the case shown, the condition that there should be at least two 'zero's between 'one's requires a 'zero' at the first merging bit position (denoted by X). There are thus only three alternatives for the merging words, namely '000', '010', and '001'. The encoder chooses the alternative that gives the lowest absolute value of the RDS at the end of a new codeword, i.e '000' in this case.

III. Description of the new method

A possible solution to the problems inherent in the prior art is based on the observation that prior art methods are overly restrictive in the choice of the merging words as only those merging words are allowed that when alternate merging words and codewords are cascaded the prescribed (d, k)-constraint is satisfied. The codewords obey both (d)- and (k)-constraint in force, but the merging words are not required to obey said d-constraint. It is known that runs of '0's smaller than d are prone to error resulting from inter symbol interference. However, as the merging words are skipped by the receiver, the (d)-constraint in the merging words can be violated without compromising the reliability of the received codewords. Then, as a result of the increase of the size of the set of merging words from which can be selected, the low-frequency components of the encoded signal can significantly be reduced with respect to signals generated under the rules of prior art.

Consecutive n-bit codewords are alternated with p-bit merging words. The encoder generates a set of allowed p-bit merging words for use between consecutive codewords so that the sequence comprised of alternate codewords and merging words satisfies the conditions that between the leading '1' in the merging word and the trailing '1' in the codeword preceding the merging word, and the trailing '1' in the merging word and the leading '1' in the codeword following the merging word are at least d '0's. In a second aspect, the encoder selects from said set of allowed merging words that merging word that yields the cumulative dc imbalance nearest zero in the catenation of the codeword and that merging word after modulo-2 integration. For example, in the EFM code, where $d = 2, k = 10, n = 14, m = 8$, and $p = 3$, there are four 3-bit merging words of the eight possible merging words allowed, namely '001', '010', '000', and '100'. By way of example, the remaining 3-bit merging words, namely '111', '011', '101', and '110' are allowed to be used. There are, however, three restrictions: firstly, in the cascade of alternate 14-bit codewords and 3-bit merging words the k(=10) constraint should not be violated, and secondly, in order to safeguard the reliability of the codewords, the number of consecutive '0's at the beginning or end of the two codewords joining the merging word should be at least d.

TABLE I
Merging operation, Example 1.

14-bit word	Merging bits	Next 14-bit word	RDS
01001000100100	000	00100000000100	4
01001000100100	001	00100000000100	10
01001000100100	010	00100000000100	8
01001000100100	011	00100000000100	2
01001000100100	100	00100000000100	6
01001000100100	101	00100000000100	0
01001000100100	110	00100000000100	2
01001000100100	111	00100000000100	8

Table I shows schematically the process of generating the set of allowed merging words, the computation of the corresponding RDS, and the selection of the merging word that yields the least dc-imbalance. We show the concatenation of the 14-bit codewords '01001000100100' and '00100000000100', where for illustrative purposes it is assumed that the RDS at the end of the first codeword '01001000100100' equals +5. For the specific case in hand all possible 3-bit merging words can be used without violating the prescribed constraints. Table I shows all possible 3-bit merging words, and it shows the resulting RDS after the merging word and codeword have been catenated and integrated modulo-2. The encoder selects that particular merging word resulting in an RDS nearest zero, i.e. it selects '101'. The RDS after cascading the merging words '101' and the codeword '00100000000100' is thus 0. In this

TABLE II
MERGING OPERATION, EXAMPLE 2.

14-bit word	Merging bits	Next 14-bit word	RDS
01001000100100	000	01000001001001	4
01001000100100	001	01000001001001	Not allowed
01001000100100	010	01000001001001	8
01001000100100	011	01000001001001	Not allowed
01001000100100	100	01000001001001	6
01001000100100	101	01000001001001	Not allowed
01001000100100	110	01000001001001	2
01001000100100	111	01000001001001	Not allowed

manner the dc-level of the encoded signal is maintained at a substantially constant level and the frequency spectrum of the encoded signal will show suppressed low-frequency components. Note that prior art EFM encoders, where the merging bits obey the prescribed (d)-constraint, will select the merging word '000', which will lead to a larger RDS, namely +4. This example shows that signals obtained in accordance with the invented method show a more constant dc-level of the encoded signal than those generated by prior art methods.

As a further illustration, Table II shows the process of generating all 3-bit merging words in the case codewords '01001000100100' and '01000001001001' are catenated. Merging words '001', '011', '101', and '111' are not allowed as the trailing '1' of the merging word and the leading '1' of the codeword '01000001001001' are less than d(=2) '0's apart. If it is assumed that the RDS at the end of the codeword '01001000100100' equals +5, then the RDS after a candidate merging word and the 14-bit codeword have been catenated and integrated modulo-2 is listed in Table II. The encoder selects the merging word resulting in the RDS nearest zero, i.e. it selects '110', so that the RDS after cascading the merging words '110' and the codeword '01000001001001' is thus +2. Note that a conventional EFM encoder, where the merging words must obey the prescribed (d)-constraint, will select the merging word '000', which will lead to a larger RDS, namely +4.

A. Results

The new coding method, as shown in Table II, may select merging words leading to a smaller RDS, which has a bearing on the low-frequency content of the encoded data stream. The spectral performance of the new method has been appraised by computer simulations, where a long series of random data was encoded using the new merging protocol. Figure 2 shows the power density function, or *spectrum*, of a long stream of random data encoded by classic EFM and that obtained by the new coding method versus the relative frequency f. Perusal of the diagram reveals that the new method improves the low-frequency content by around 4 dB.

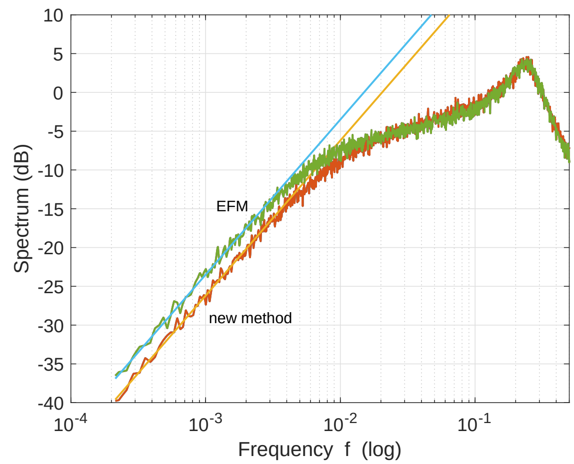

Fig. 2. Spectrum of classic EFM and the new method (dB) versus relative frequency f (log axis).

IV. CONCLUSIONS

We have presented a new coding method used for recording and transmission an audio or video signal, computer data, etc on a recording medium or by visible light. Data words of m bits are translated into codewords of n bits in accordance with a prescribed conversion table. The n-bit codewords satisfy a (d, k)-constraint in which at least d '0's and not more than k '0's occur between consecutive '1's. The n-bit codewords alternate with p-bit merging words which do not need to satisfy the (d)-constraint. They are selected such that between the leading '1' in the codeword following the merging word and the trailing '1' in the merging word are at least d '0's, and further that between the trailing '1' in the codeword preceding the merging word and the leading '1' in the merging word are at least d '0's. The merging word that satisfies said conditions, yielding the lowest dc imbalance of the encoded signal obtained after modulo-2 integration of the catenation of the alternate codewords and merging words is selected. The spectral performance of the new method has been appraised by computer simulations for the EFM (Eight-to-Fourteen Modulation) parameters, namely $d = 2, k = 10, p = 3$, where a long series of random data was encoded using the new merging protocol. The low-frequency content of the signal generated by the newly presented method is around 4 dB lower in the relevant low-frequency range than that of the conventional EFM method.

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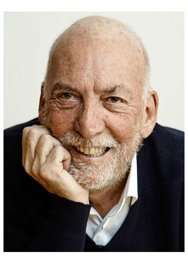

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