Coding Schemes for Multi-Level Channels with Unknown Gain and/or Offset

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Summary - *We will present coding techniques for transmission and storage channels with unknown gain and/or offset. It will be shown that a codebook of length-n q-ary codewords, S, where all codewords in S have equal balance and energy show an intrinsic resistance against unknown gain and/or offset. Generating functions for evaluating the size of S will be presented. We will present an approximate expression for the code redundancy for asymptotically large values of n.*

Key words: Constant composition code, permutation code, flash memory, optical recording

I. Introduction

We consider a communication codebook, S, of chosen q-ary sequences $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ over the q-ary alphabet $\mathcal{Q} = \{0, 1, \ldots, q-1\}$, $q \geq 2$, where n, the *length* of $\mathbf{x}$, is a positive integer. Usually it is assumed that the primary distortion of a sent codeword $\mathbf{x}$ is additive noise. Here, however, it is assumed that the received vector, $\boldsymbol{r}$, where $\boldsymbol{r} = a(\mathbf{x} + \boldsymbol{\nu}) + b$, $a > 0$, is scaled by an unknown positive scaling factor (gain), a, and offsetted by an unknown offset, b, (that is, both quantities are unknown to both sender and receiver), where a and b are real numbers, and corrupted by additive noise $\boldsymbol{\nu} = (\nu_1, \ldots, \nu_n)$.

Examples of such channels in practice are optical disc, where the gain and offset depend on the reflective index of the disc surface and the dimensions of the written features [1]. Reading errors in solid-state (Flash) memories may originate from low memory endurance, by which a drift of threshold levels in aging devices may cause programming and read errors [2], [3].

Mismatch between receiver and channel may lead to serious degradation of the error performance. There have been a few proposals to improve the detection process by making it less dependent on the channel's actual gain and offset. We may introduce automatic gain (AGC) and offset control, which depends on the weighted average gain and offset of previously received codewords. In most channels, gain and offset are time-variant so that the gain and offset control may be suboptimal. We may use parts of the memory cell array as reference cells. The reference cells are written with known signal levels, and are continuously monitored to obtain estimates of the channel's momentary gain and offset. The estimated offset values will be used to calibrate the threshold levels. In addition, codes have been used in practice to make the detection quality less dependent of the channel's gain and offset. In optical disc recording devices, *dc-balanced* binary codes have been used to counter the effects of unknown gain and offset levels [4]. Jiang *et al.* [2] addressed a coding technique called *rank modulation* for circumventing the difficulties in flash memories having aging offset levels.

In this paper, we will study maximum likelihood detection of q-ary codewords stored in a solid-state memory whose gain and/or offset may change over time, and are assumed to be unknown to the receiver. We will investigate the properties of codewords that will make them immune against uncertainty in the channel's gain and/or offset. Two quantities, as we will show later, play a key role in the design of such intrinsic resistance codes, namely the *balance*

$$d_c(\boldsymbol{x}) = \sum_{i=1}^{n} x_i,$$

and the *energy*

$$d_p(\boldsymbol{x}) = \sum_{i=1}^{n} x_i^2,$$

which depend on the codeword $\boldsymbol{x}$. We will consider codes, S, where $d_c(\boldsymbol{x})$ and $d_p(\boldsymbol{x})$ are constant with respect to $\boldsymbol{x}$ in S.

In Section II, we will show that codes consisting of codewords that satisfy a balance (B), or energy (E), or a combination of both (BE) constraints, are intrinsically resistant against the channel's unknown gain and/or offset. In Section III, we will study properties of the set S of constrained codewords. We will, in Sections IV and V, compute the size of S using generating functions, and find, using statistical arguments, an approximate expression of the redundancy of BE constrained channels for asymptotically large values of n. In Section VI, we will describe our conclusions.

II. Intrinsically resistant codes

We select a codebook, S, of codewords, $\mathbf{x}$, that all satisfy the same *(E)nergy* constraint

$$\sum_{i=1}^{n} x_i^2 = d_p,$$

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and all satisfy the same *(B)alance* constraint

$$\sum_{i=1}^{n} x_i = d_c,$$

where d_c and d_p are predefined positive integers. The integers d_c and d_p are not of interest at the moment, but, clearly, a specific choice has a bearing on the size of S.

Assume the sequence $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ over the q-ary alphabet $\mathcal{Q} = \{0, 1, \ldots, q-1\}$ is sent to the receiver. The received word, $\boldsymbol{r} = a(\mathbf{x} + \boldsymbol{\nu}) + b$, is scaled by an unknown scaling factor (gain), a, offsetted by an unknown offset b, and distorted by additive noise $\boldsymbol{\nu}$. Applying maximum-likelihood (ML) detection, the receiver computes for each $\hat{\mathbf{x}} \in S$ the Euclidean distance

$$d = \sum_{i=1}^{n} (r_i - \hat{x}_i)^2 = \sum_{i=1}^{n} (ax_i + b + a\nu_i - \hat{x}_i)^2 \qquad (1)$$

between the received signal $\boldsymbol{r}$ and $\hat{\mathbf{x}}$. After computing the metric d for all $\hat{\mathbf{x}} \in S$, the receiver decides that the codeword $\hat{\mathbf{x}}_{\mathbf{o}}$ with minimum distance to the received vector, $\boldsymbol{r}$, is the codeword sent. After working out (1), we obtain

$$\begin{aligned} d &= \sum_{i=1}^{n} (ax_i + b + a\nu_i)^2 - 2\sum_{i=1}^{n} (ax_i + b + a\nu_i)\hat{x}_i + \sum_{i=1}^{n} \hat{x}_i^2 \\ &= \sum_{i=1}^{n} (ax_i + b + a\nu_i)^2 - 2a\sum_{i=1}^{n} (x_i + \nu_i)\hat{x}_i - 2bd_c + d_p. \end{aligned}$$

The first term and the quantity $-2bd_c + d_p$ are both independent of $\hat{\boldsymbol{x}}$ and by pre-assumption $a > 0$, the decoder seeks to find that vector $\hat{\boldsymbol{x}} \in S$ which minimizes

$$d = \sum_{i=1}^{n} (r_i - \hat{x}_i)^2 = -\sum_{i=1}^{n} (x_i + \nu_i)\hat{x}_i + \psi, \qquad (2)$$

where ψ is a term independent of $\hat{\boldsymbol{x}}$. As desired, the distance metric (2) is now intrinsically resistant against the channel's gain and offset, since the gain and offset terms, a and b, are absent in the metric (2). In case of unknown offset only, i.e. $a = 1$, we may choose a codebook whose codewords satisfy a balance (B) constraint only, and in case the offset is known, $b = 0$, we may choose a codebook with an energy (E) constraint only.

In the next sections, we will study properties of the codebook S whose codewords satisfy the equal balance (B), or energy (E), or both (BE) constraints. Specifically, we will study how we can maximize the codebook size by a judicious choice of the parameters d_c and/or d_p.

III. Selecting the Codewords

A *constant composition* code of n-length codewords over a q-ary alphabet has the property that the numbers of occurrences of the q symbols within a codeword is the same for each codeword [5]. These specialize to constant weight codes in the binary case, and permutation codes in the case that each symbol occurs exactly once. Define the *composition vector* $\boldsymbol{w}(\boldsymbol{x}) = (w_0, \ldots, w_{q-1})$ of $\boldsymbol{x}$, where the q entries w_i, $i \in \mathcal{Q}$, of $\boldsymbol{w}$ indicate the number of occurrences of the symbol $x_i \in \mathcal{Q}$ in $\boldsymbol{x}$, that is for a q-ary sequence $\mathbf{x}$, we denote the number of appearances of the symbol j by

$$w_j(\mathbf{x}) = |\{i : x_i = j\}| \text{ for } j = 0, 1, \ldots, q-1. \qquad (3)$$

Clearly $\sum w_i = n$ and $w_i \in \{0, \ldots, n\}$. A constant composition code will be denoted by $S_{\boldsymbol{w}}$, where $\boldsymbol{w}$ is the composition vector that characterizes the code. The size of $S_{\boldsymbol{w}}$ equals

$$|S_{\boldsymbol{w}}| = \frac{n!}{\prod_{i \in \mathcal{Q}} w_i!}. \qquad (4)$$

Define the first three moments $M_{\boldsymbol{w}} = (m_0, m_1, m_2)$ of the composition vector $\boldsymbol{w}$ by

$$m_0 = \sum_{i \in \mathcal{Q}} w_i = n,$$

$$m_1 = \sum_{i \in \mathcal{Q}} i w_i,$$

and

$$m_2 = \sum_{i \in \mathcal{Q}} i^2 w_i.$$

It is immediate that codewords in a constant composition code satisfy the BE constraint, where $d_c = m_1$ and $d_p = m_2$. Such codes are known as *permutation modulation* codes (Variant I), and were introduced by Slepian [6] in 1965.

While Slepian's coding scheme, variant 1, uses a single constant composition code, other coding schemes can be based on a plurality of constant composition codes. In particular, the set of codewords defining the codebook S can be selected from the codewords of a plurality of constant composition codewords codes so as to satisfy the equal balance and energy requirement. Therefore, we investigate a codebook S, which is the union of a plurality K, $K \geq 1$, of such constant composition codes. Denote the constant composition codes by $S_{\boldsymbol{w}_i}$, $1 \leq i \leq K$, then we have

$$S = \bigcup_{i=1}^{K} S_{\boldsymbol{w}_i}.$$

Thus, in summary, we must try to find K composition vectors $\boldsymbol{w}_i$, $1 \leq i \leq K$, over the alphabet $\{0, \ldots, n\}$ that all satisfy

$$M_{\boldsymbol{w}_i} = (n, d_c, d_p),$$

and maximize the code size $|S| = \sum_i |S_{\boldsymbol{w}_i}|$ by a proper choice of the parameters d_c and d_p. The above conditions are reminiscent of the conditions imposed on binary codewords that satisfy higher-order spectral constraint [7]. In the next section, we will show that we can find the K subsets that satisfy the constant balance and energy constraint using generating functions [9].

IV. Counting using generating functions

We will use *generating functions* for enumerating the number of sequences $\mathbf{x}$ that satisfy the equal-balance-energy (BE) constraints. Let the generating function $g(y)$ be defined by the formal power series

$$g(y) = \sum_{i=0}^{\infty} g_i y^i.$$

We let $[y^n]g(y)$ denote the *extraction* of the coefficient of y^n in the formal power series $g(y) = \sum g_i y^i$, that is, define

$$[y^n]\left(\sum g_i y^i\right) = g_n.$$

In a similar fashion, we define a bivariate generating function

$$\hat{g}(x, y) = \sum_{i,j} \hat{g}_{i,j} x^i y^j.$$

The number of sequences that satisfy a given balance, d_c, and energy constraint, d_p, is given by $[x^{d_c} y^{d_p}]g_q(x, y)$, where

$$g_q(x, y) = \left(\sum_{i \in \mathcal{Q}} x^i y^{i^2}\right)^n. \tag{5}$$

The number of sequences, N_B, that satisfy a given balance constraint d_c is given by

$$N_B = [x^{d_c}]g_q(x, y = 1),$$

and the number of sequences, N_E, that satisfy a given energy constraint d_p is given by

$$N_E = [y^{d_p}]g_q(x = 1, y).$$

By a judicious choice of the parameters d_c or d_p, we can maximize the code sizes N_B and N_p, respectively. The above generating functions enumerates the numbers of codewords. We can use the enumeration of composition vectors, and obtain an alternative generating function, namely

$$f(x, y, z) = n! \prod_{j=0}^{q-1} \sum_{i=0}^{n} \frac{1}{i!} x^i y^{ij} z^{ij^2}, \tag{6}$$

where the coefficient $x^u y^v z^w$ of $f(x, y, z)$ equals the number of codewords $\mathbf{x}$ whose composition vector $\mathbf{w}(\mathbf{x})$ satisfies

$$\sum_{i \in \mathcal{Q}} w_i = u, \sum_{i \in \mathcal{Q}} i w_i = v, \sum_{i \in \mathcal{Q}} i^2 w_i = w.$$

As we are interested in composition vectors that satisfy $\sum_{i \in \mathcal{Q}} w_i = n$, we define

$$\phi(y, z) = [x^n]f(x, y, z).$$

The maximum size of the BE-constrained codebook, S, denoted by $N_{BE}(n)$, equals the largest coefficient of $\phi(y, z)$, that is

$$N_{BE}(n) = \max_{i,j}[y^i z^j]\phi(y, z), \tag{7}$$

where $N_{BE}(n)$ denotes the number of n-length q-ary sequences that satisfy the BE-constraint. Let i_0 and j_0 denote the integers that maximize the above expression. Then, the number of subsets, K, that constitute S, can be found be extracting the coefficient $[x^n y^{i_0} z^{j_0}]f'(x, y, z)$, where

$$f'(x, y, z) = \prod_{j=0}^{q-1} \sum_{i=0}^{n} x^i y^{ij} z^{ij^2}. \tag{8}$$

In a similar fashion, we define $N_B(n)$ and $N_E(n)$ for the number of B- and E-constrained sequences, respectively. In the next section, we will use (7) for computing the size of various constrained codebooks.

TABLE I
MAXIMUM SET SIZE $N_{BE}(q)$ AND RATE R_{BE} VERSUS q. THE QUANTITY K DENOTES THE NUMBER OF CONSTANT COMPOSITION CODES.

$q = n$	$N_{BE}(q)$	R_{BE}	K
4	24	0.573	1
5	120	0.595	1
6	720	0.612	1
7	6300	0.642	3
8	68320	0.669	10
9	945000	0.696	26
10	15766380	0.720	67
11	302577660	0.740	182
12	6550665264	0.758	522

TABLE II
MAXIMUM SET SIZE $N_E(q)$ AND RATE R_E VERSUS q. THE QUANTITY K DENOTES THE NUMBER OF CONSTANT COMPOSITION CODES.

$q = n$	$N_E(q)$	R_E	K
4	24	0.573	1
5	130	0.605	2
6	1140	0.655	4
7	11655	0.687	14
8	160217	0.720	41
9	2603073	0.747	108
10	49756440	0.770	318

A. The binary case

For the binary case $q = 2$, the energy d_p and balance d_c of each binary sequence equals the number of 1's in the codeword, and the number of sequences with balance d_c equals the binomial coefficient

$$N_2(n) = \binom{n}{d_c}.$$

For example, let $n = 4$, and let the set of 'dc-balanced' codewords consist of all words that have two '1's and two '0's. Let the received vector be $\boldsymbol{r}$ = (1.2, 2.3, 0.4, 0.8). The detector assigns the two lowest levels to the '0's and the two highest levels to the 1's [6]. Thus the decoder finds (1,1,0,0) as the codeword sent. We can easily verify that the above detection process of finding peaks and dales is intrinsically resistant against any offset or gain mismatch.

B. The non-binary case, $n = q$

For $n = q$, using (7) and a symbolic manipulation system, we computed $N_{BE}(q)$. For $q(= n) < 7$ we find $N_{BE}(q) = q!$,

TABLE III
MAXIMUM SET SIZE $N_B(q)$ AND RATE R_B VERSUS q. THE QUANTITY K DENOTES THE NUMBER OF CONSTANT COMPOSITION CODES.

$q=n$	$N_B(q)$	R_B	K
4	44	0.682	5
5	381	0.738	12
6	4332	0.779	32
7	60691	0.808	94
8	1012664	0.831	289
9	19610233	0.849	910

TABLE IV
COMPOSITION VECTORS OF THE CODING SUBSETS S_i, $i \in \{1,2,3\}$, FOR $q=n=7$.

i	$w_{i,0}$	$w_{i,1}$	$w_{i,2}$	$w_{i,3}$	$w_{i,4}$	$w_{i,5}$	$w_{i,6}$
1	1	1	1	1	1	1	1
2	2	0	0	1	2	2	0
3	0	2	2	1	0	0	2

while for $q = n > 6$ we have $N_{BE}(q) > q!$. Results of computations for the case $q = n > 3$ are collected in Table I, where the code rate, R_{BE}, is defined by

$$R_{BE} = \frac{1}{q}\log_q N_{BE}(q).$$

The quantity, K, the number of constant composition codes in S, was computed by evaluating generating function (8).

Example 1 Let $q = n = 7$. The maximum codebook S for a BE-constrained comprises three constant composition codes, whose composition vectors are listed in Table IV. We can easily check that, for each permutation set, the sum of the symbol values and the sum of the squares of the symbol values equals 21 and 91, respectively. The size of the set S is the sum of the sizes of the three subsets, which can easily be computed from their composition vectors

$$N_{BE}(7) = 7! + \frac{7!}{2!1!2!2!} + \frac{7!}{2!.2!.1!.2!} = 6300.$$

In this instance, the codebook S comprises the permutation (sub)set with unity composition vector. Note, however, in the code sets that we have investigated this is more the exception than the rule.

Results of computations of $N_E(q)$, $N_B(q)$, etc are listed in Tables II and III.

C. The non-binary case, $n \neq q$

V. ASYMPTOTICS

Using statistical arguments, we will compute an approximation to the redundancy of BE constrained channels for asymptotically large values of n. Define

$$c = x_1 + x_2 \ldots + x_n,$$

and

$$p = x_1^2 + x_2^2 \ldots + x_n^2,$$

where x_i are random variables in $\mathcal{Q}$. In case n is large, we may expect that the stochastic variables c and p, which are obtained by summing n independent stochastic variables, will show a Gaussian distribution. Then, for asymptotically large n, the number of n-sequences, $N_{BE}(n)$, has the two-dimensional normal distribution

$$N_{BE}(n) \approx \frac{q^n}{\pi\sigma_c\sigma_p\sqrt{1-\rho^2}} e^{-\frac{1}{2(1-\rho^2)}f(c,p)},$$

where

$$f(c,p) = (\frac{c-\mu_c}{\sigma_c})^2 + (\frac{p-\mu_p}{\sigma_p})^2 - \frac{2\rho(c-\mu_c)(p-\mu_p)}{\sigma_c\sigma_p}.$$

The minimum redundancy, $r_{BE}(n)$, for asymptotically large n is, by choosing $c = \mu_c$ and $p = \mu_p$, given by

$$\begin{aligned} r_{BE}(n) &= n - \log_q N_q(n) \\ &\approx \log_q \pi\sigma_c\sigma_p\sqrt{1-\rho^2}. \end{aligned} \quad (9)$$

The parameters μ_c, μ_p, etc. can be found by a manipulation of the moments

$$e_j = E[x_i^j] = \frac{1}{q}\sum_{i\in\mathcal{Q}} i^j,\ j = 1,\ldots,4.$$

After working out, we have

$$\mu_c = ne_1 = \frac{q-1}{2}n,$$

$$\sigma_c^2 = n(e_2 - e_1^2) = \frac{q^2-1}{12}n,$$

$$\mu_p = ne_2 = \frac{(q-1)(2q-1)}{6}n,$$

$$\begin{aligned} \sigma_p^2 &= n(e_4 - e_2^2) \\ &= \frac{(q^2-1)(8q-11)(2q-1)}{180}n \end{aligned}$$

and

$$\rho^2 = \left(\frac{n(e_3 - e_1e_2)}{\sigma_c\sigma_p}\right)^2 = \frac{15(q-1)^2}{(8q-11)(2q-1)}.$$

So that we obtain, using (9),

$$\begin{aligned} r_{BE}(n) &\approx \log_q \pi\sigma_c\sigma_p\sqrt{1-\rho^2} \quad (10) \\ &= \log_q n + \log_q(q^2-1)\sqrt{q^2-4} + \log_q \frac{\pi}{12\sqrt{15}}. \end{aligned}$$

In a similar fashion, we can derive approximation to $r_B(n)$ and $r_E(n)$ for asymptotically large values of n. We find

$$\begin{aligned} r_B(n) &\approx \log_q \sigma_c\sqrt{2\pi} \\ &= \frac{1}{2}\log_q n + \frac{1}{2}\log_q(q^2-1) + \frac{1}{2}\log_q \frac{\pi}{6} \end{aligned} \quad (11)$$

and

$$r_E(n) \approx \log_q \sigma_p\sqrt{2\pi}. \quad (12)$$

The result for $r_B(n)$ is in accordance with result derived in [10] and [11]. The numerical accuracy of (10) was assessed. The results in Table I obtained show for $q = n > 7$ that

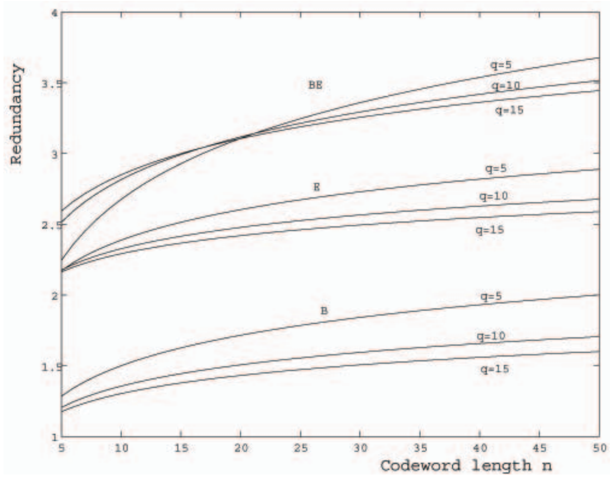

Fig. 1. Redundancy of B-, E-, and BE-constrained codes versus codeword length n with q as a parameter.

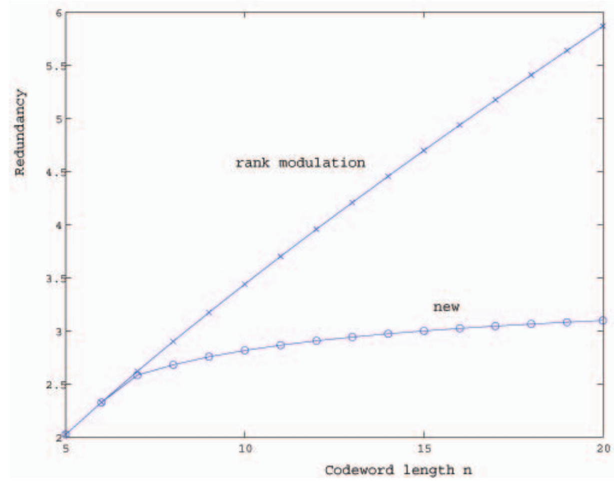

Fig. 2. Redundancy of rank modulation and BE constrained codes versus codeword length n ($q = n$).

the redundancy computed using generating functions is quite close, within 0.1%, to the redundancy approximation above using a Gaussian distribution. In Figure 1, we have plotted $r_{BE}(n)$, $r_E(n)$, and $r_B(n)$ versus n with q as a parameter. Immink [12] presented a simple routine for generating B-constrained q-ary codes whose redundancy is approximately $\log_q n$, $n >> 1$.

Prior art systems use a single permutation code, where the symbols of each codeword of length $n = q$ are a permutation of all symbols in $\mathcal{Q}$. The maximum code size equals $q!$. Using Stirling's approximation

$$q! \approx \sqrt{2\pi q}\left(\frac{q}{e}\right)^q,$$

the redundancy of a permutation code, $r_f(q)$, is

$$r_f(q) = q - \log_q(q!) \approx q\log_q(e) - \frac{1}{2}\log_q(2\pi) - \frac{1}{2}. \quad (13)$$

In Figure 2, we have plotted the redundancy of rank modulation (13) and BE-constrained codes (10) versus codeword length n, where $q = n$. We conclude that the usage of a code with multiple permutation may significantly reduce the coding redundancy.

VI. Conclusions

We have presented codes for channels with unknown gain, a, and/or offset, b, where a sent codeword $\boldsymbol{x}$ is received as $a(\boldsymbol{x} + \boldsymbol{\nu}) + b$. We have defined a codebook, S, consisting of n-length q-ary codewords, where all codewords in S have equal balance and equal energy. We have shown that maximum likelihood detection of codewords taken from S is intrinsically resistant against unknown gain and/or offset. We have presented generating functions for enumerating the number of codewords with a prescribed balance and/or energy (BE) constraint. Using statistical arguments, we have developed an approximate expression for the redundancy of BE-constrained channels for asymptotically large values of the codeword length n.

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