Error Correction Code Failure Rate Analysis for Parity-Check-Coded Optical Recording Systems

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1. Introduction

In data storage systems, the error correction code (ECC) failure rate (EFR) serves as the ultimate measure of the data recovery performance. There have been several papers on post-ECC analysis of the parity-check (PC)-code-based detection approaches for magnetic recording systems [1,2]. However, for optical recording systems whose ECC configurations as well as code constraints [3] differ significantly from those of magnetic recording systems, no report has been found on the analysis of EFR.

In this paper, we propose a generalized block multinomial method for estimating the failure rate of interleaved ECC, for constrained codes with PC. We further analyze the EFRs of a complete PC-coded optical recording system. A block diagram of a PC-code-based optical recoding system is shown in Fig. 1. The d=1 constrained PC code is an inner ECC, which can detect dominant short error events of the system, using only a few parity bits. The task of locating the exact positions of the errors is done by a post-processor. This reduces the loss in error correction capability of the outer ECC due to random short errors and results in a simple and efficient solution to improve the overall performance.

2. Generalized block multinomial method

For data storage systems, the EFR is typically in the range of 10^{-12} ~10^{-15}, and this makes the estimation of EFR by the direct count approach impractical. Several analytical approaches have been developed for estimating the ECC's failure rate [1,2], among which the multinomial model proposed in [1] has been found to be simple and efficient. In this model, it is assumed that consecutive byte errors at the ECC decoder input are caused by independent error events. Therefore, the number of consecutive byte errors in a received ECC codeword can be modeled by a multinomial distribution. However, for systems with PC codes and post-processing, such an assumption may not be accurate, since it does not account for the dependence of byte errors due to the mis-correction of the post-processor. In [2], a block multinomial model is proposed for PC-coded systems with non-interleaved ECC, which has been shown to provide better EFR estimation than the multinomial model, for PC-coded magnetic recording systems.

Interleaving is a useful tool to enhance the capability of ECC for correcting burst errors [3], and it is widely used in error correction coding for optical recording systems. In this part, we develop a generalized block multinomial method for estimating the failure rate of interleaved ECC. The failure rate of interleaved ECC corresponds to the probability that at least one of the ECC codewords in the interleaved codewords array has more than t byte errors, where t is the number of byte errors that the ECC can correct. Let N_i denote the number of byte errors in the i^{th} interleave. In the presence of interleaving to a finite degree of α, the EFR, P_{If}, can be computed as $P_{If} \le \sum_{i=1}^{\alpha} P_f(i)$, where $P_f(i) = \Pr\{N_i > t\}$, for $1 \le i \le \alpha$.

In the proposed generalized block multinomial method, we define byte errors on a PC codeword basis, and consecutive PC codewords within each interleave are considered to be independent. That is, we define a polynomial

$$q(D) = q_1 D + q_2 D^2 + \cdots + q_k D^k + \cdots + q_r D^r, \tag{1}$$

where q_k is the probability of receiving k byte errors in a PC codeword within each interleave, and r is the maximum number of byte errors within an interleaved PC codeword. The EFR $P_f(i)$ for the ECC in each interleave is then given by

$$P_f(i) = \sum_{j \ge 1}^{\min(n_I, j_{\max})} q_0^{n_I - j} \cdot q_{j,t+1} \cdot \frac{n_I!}{j!(n_I - j)!}, \tag{2}$$

where $q_{j,t+1}$ is the probability of having j error events which lead to t+1 or more byte errors in the ECC codeword within each interleave, and $j_{\max}$=2t is a truncation parameter, beyond which $q_{j,t+1}$ is negligible. Here, n_I is the number of PC codewords in the ECC codeword within each interleave, and $q_0 = 1 - \sum_{j=1}^{r} q_j$.

3. Simulation results and discussion

We first verify the above proposed generalized block multinomial method. We assume an idealized blue laser disc system with a high recording density [4]. Furthermore, a rate 277/406 4-bit constrained PC code proposed in [4], together with a Reed-Solomon (RS) [248, 216] code with 8 bits per symbol (byte) [3] is used in the simulations. With an α=5-degree interleaving, the comparison between EFRs obtained from the direct count approach and those obtained from the generalized method is shown in Fig. 2. Observe that with signal-to-noise ratios (SNRs) higher than 13 dB, the generalized block multinomial method provides very accurate estimates on the EFRs. With SNRs lower than 13 dB, the EFRs obtained from the generalized method have a bias of less than 0.1 dB, at EFR=10^{-1}~10^{0}. This is because when the SNR is very low, there exists simultaneously more than *t* byte errors over different interleaves, and they are ignored by the analytical approach. However, as SNR increases, the occurrence of such byte errors decreases significantly, and becomes negligible for SNRs higher than 13 dB.

Using the proposed generalized block multinomial method, we analyze the EFRs of the PC-coded system described in Section 1. The study also includes a rate 9/13 *d*=1 code, whose code rate is around 3.85% higher than that of the standard codes [3] used for blue laser disc systems. The EFRs of the rate 9/13 code are estimated using the multinomial method [1]. Fig. 3 illustrates the EFRs obtained for both cases without and with interleaving. For the case with interleaving, the optimum interleaving degrees are used. They are taken as α=5 and α=36, respectively, for the rate 9/13 code and the 4-bit PC code. The figure shows that in the case of without interleaving, the performance of the 4-bit PC code lags behind the rate 9/13 code. This is mainly due to the long byte errors introduced by post-processing. Through interleaving, these long byte errors have been effectively broken into short byte errors over different interleaves. This enhances the error correction capability of the RS-ECC, and results in a larger gain for the 4-bit PC code. In the case of with interleaving, the 4-bit PC code achieves around 0.6 dB improvement over the rate 9/13 code, at EFR=10^{-15}. The EFR gain over the standard codes [3] will be even more significant.

4. Conclusions

In this paper, we have proposed a generalized block multinomial method for estimating the failure rate of interleaved ECC, for constrained codes with PC. We have further evaluated the EFRs of a PC-coded optical recording system. The analysis shows that with optimum interleaving degrees, the 4-bit constrained PC code achieves a gain of 0.6 dB over the rate 9/13 code without PC, at EFR=10^{-15}.

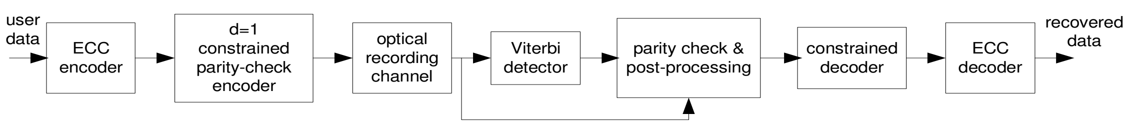

Fig. 1. Block diagram of a PC-coded optical recording system.

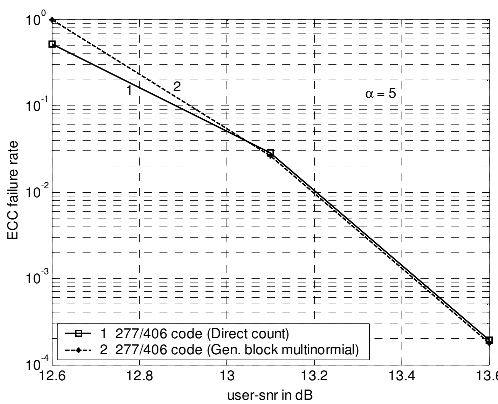

Fig. 2. Comparison of EFR evaluation methods.

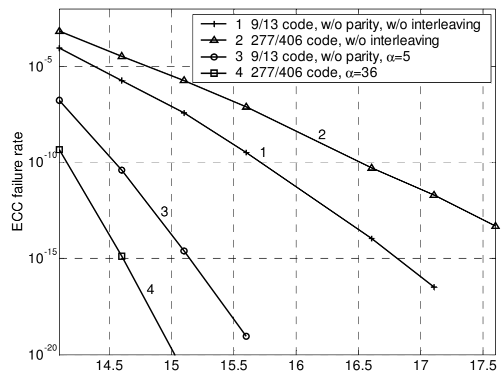

Fig. 3. EFR comparison with various codes.

References

1. W. Feng *et al.*, "On the performance of parity-check codes in magnetic recording systems," *Proc.* GLOBECOM, San Francisco, USA, Nov. 2000, pp. 1877-1881.
2. Z.A. Keirn *et al.*, "Use of redundant bits for magnetic recording: single-parity codes and Reed-Solomon error-correcting code, " *IEEE Trans. Magnetics*, vol. 40, no. 1, pp. 225-230, Jan. 2004.
3. T. Narahara *et al.*, "Optical disc system for digital video recording," *JJAP*, pt. 1, vol. 39, No. 2B, pp. 912-919, 2000.
4. K. Cai *et al.*, "Constrained parity-check code and post-processor for advanced blue laser disc systems," *JJAP*, vol. 45, No. 2B, pp. 1071-1078, 2006.