Coding Schemes for Multi-Level Flash Memories that are Intrinsically Resistant Against Unknown Gain and/or Offset Using Reference Symbols

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We will present coding schemes for storage channels, such as optical recording and Non-Volatile Memory (Flash), with unknown gain and offset. In its simplest case, the coding schemes guarantee that a symbol with a minimum value (floor) and a symbol with a maximum (ceiling) value are always present in a codeword so that the detection system can estimate the momentary gain and offset. Results of computer simulations will show the performance of the new coding and detection methods in the presence of additive noise.

Introduction: In practical storage and communication systems it is usually found that noise is an important issue but that also other important physical facors may hamper the reliability of tranmission or storage systems. For example, reading errors in solid-state (Flash) memories may originate from low memory endurance, by which a drift of charge levels in aging memory cells may cause programming and read errors [1]. In optical disc recording, the signal strength (gain) and signal offset depend on the reflective index of the disc surface and the dimensions of the written features [2]. Fingerprints on the disc may result in rapid gain and offset variations of the retrieved signal.

The gain and offset uncertainties may lead to massive performance degradation as shown, for example, in [3]. In the prior art, data reference, 'training', patterns are multiplexed with the user data in order to 'teach' the data detection circuitry the momentary values of the channel's characteristics such as impulse response, gain, and offset. In a channel with unknown gain and offset, we may use two reference symbol values, where in each codeword, a first symbol is set equal to the lowest signal level and a second symbol equal to the highest signal level. The positions and amplitudes of the two reference symbols are known to the receiver. The receiver can measure the amplitude of the retrieved reference symbols, and normalize the amplitudes of the remaining symbols of the retrieved codeword before applying threshold detection.

Clearly, the redundancy of the method is two symbols per codeword. We may reduce the redundancy by noticing that it is sufficient that each codeword contains at least one symbol with a lowest level and one symbol with a highest level, where the positions of the two reference symbols are not *a-priori* known to the receiver. Then, the receiver can rank the retrieved symbols of the codeword, and it computes the minimum and maximum value of the retrieved signal values, which provides sufficient information for estimating the channel's offset and gain. Coding techniques are needed to guarantee that each codeword has at least one symbol with a lowest level and one symbol with a highest level. To that end, we will introduce T-constrained codes comprising codewords, where T *preferred* or *reference* symbols must appear at least once in a codeword. The receiver using a threshold detector may exploit the reference symbols for calibrating its threshold levels by estimating the lowest and highest retrieved signal values, so that the receiver can fruitfully counter the detrimental effects of the channel's unknown gain and offset. We will study the performance of the new coding and detection method assuming the presence of additive Gaussian noise.

Codewords with T reference symbols: We consider a communication codebook, S, of chosen q-ary sequences $\boldsymbol{x} = (x_1, x_2, \ldots, x_n)$ over the q-ary alphabet $\mathcal{Q} = \{0, 1, \ldots, q-1\}$, $q \geq 2$, where n, the *length* of $\boldsymbol{x}$, is a positive integer. Usually it is assumed that the primary distortion of a sent codeword $\boldsymbol{x}$ is additive noise, but here, however, it is assumed that the retrieved signal vector, $\boldsymbol{r}$, $\boldsymbol{r} = a(\boldsymbol{x} + \boldsymbol{\nu}) + b$, is scaled by an unknown scaling factor (gain), a, and off set by an unknown offset b (unknown to both sender and receiver), where a and b are real numbers, and corrupted by additive noise $\boldsymbol{\nu} = (\nu_1, \ldots, \nu_n)$. We will now address the problem of counting n-length q-ary sequences, where in each sequence $\boldsymbol{x} = (x_1, x_2, \ldots, x_n)$ T, $T \leq q$, specific reference symbols must appear at least once. Clearly, the counting of sequences is not affected by the choice of the specific T symbols we like to favor. In order to reduce clerical work, we assume therefore, without loss of generality, that the symbols with values in $\{0, \ldots, T-1\}$ must appear at least once in $\boldsymbol{x}$. Thus each codeword, $(x_1, x_2, \ldots, x_n)$, in a T-constrained code, has the property that

$$|\{i : x_i = j\}| > 0 \text{ for } j = 0, 1, \ldots, T-1.$$

We are now in the position to write down an expression for the number of T-constrained sequences.

Theorem 1: The number of n-length q-ary sequences, $N_T(n)$, where T, $T \leq q$, distinct symbols occur at least once in the sequence, equals

$$N_T(n) = \sum_{i=0}^{T} (-1)^i \binom{T}{T-i} (q-i)^n, \ n \geq T. \tag{1}$$

Proof. To solve this problem, we use the inclusion-exclusion principle to find all T-constrained q-ary n-length codewords. Clearly, there are q^n distinct q-ary codewords of length n. We then subtract and add (T choose $T-i$) $(q-i)^n$ repeatedly until we have counted all possible codewords, which is a standard application of the principle of inclusion-exclusion and is exactly what the theorem states. This gives us the explicit formula for $N_T(n)$. ■

The number of n-length q-ary sequences, $N_1(n)$, where $T = 1$ out of the q possible symbols occur at least once, equals

$$N_1(n) = q^n - (q-1)^n, q > 1. \tag{2}$$

The number of n-length q-ary sequences, $N_2(n)$, where $T = 2$ reference symbols occur at least once, equals

$$N_2(n) = q^n - 2(q-1)^n + (q-2)^n, q > 1. \tag{3}$$

For the binary case, $q = 2$, we simply find for $T = 1$ that

$$N_1(n) = 2^n - 1$$

(the all-'1' word is deleted), and for $T = 2$ we have

$$N_2(n) = 2^n - 2$$

(both the all-'1' and all-'0' words are deleted). T-constrained codes can efficiently be implemented by a generalization of the binary nibble replacement technique [4] to q-ary schemes.

Dynamic threshold detection: We assume that the codeword $\boldsymbol{x}$ is stored. The retrieved word, $\boldsymbol{r} = a(\boldsymbol{x} + \boldsymbol{\nu}) + b$, where $\boldsymbol{r}$ is an n-vector of reals, is scaled by an unknown scaling factor (gain), a, off set by an unknown offset, b, and corrupted by additive noise $\boldsymbol{\nu} = (\nu_1, \ldots, \nu_n)$, where ν_i are noise samples with distribution $N(0, \sigma^2)$. The retrieved word, $\boldsymbol{r}$, can be straightforwardly detected (quantized) with a threshold detector with fixed thresholds. Since the reference symbols '0' and '$q-1$' appear at least once, we may estimate the channel's offset, b, and gain, a, as follows. The symbols of the retrieved word $\boldsymbol{r}$ are sorted, largest to smallest, then estimates of the gain $\hat{a}$ and the offset $\hat{b}$ are given by

$$\hat{a} = \frac{\max\{r_i\} - \min\{r_i\}}{q-1} \tag{4}$$

and

$$\hat{b} = \min\{r_i\}. \tag{5}$$

After normalizing the retrieved signal vector by $\hat{a}$ and $\hat{b}$, i.e.

$$\boldsymbol{r}' = \frac{\boldsymbol{r} - \hat{b}}{\hat{a}},$$

the normalized vector, $\boldsymbol{r}'$, is in its standard range, and detection is straightforward. Note that both estimators $\hat{a}$ and $\hat{b}$ are biased. A codeword will contain $L_0 \geq 1$ '0's and $L_{q-1} \geq 1$ '$q-1$' symbols, so that $\hat{b} = \min\{r_i\}$ is the infimum of the set of L_0 noise samples. In the next subsection, we will appraise the noise margin of the dynamic threshold detection method by presenting results of computer simulations.

Simulations

We simulated the performance of the system by assuming additive zero-mean Gaussian noise with variance σ^2. The signal-to-noise ratio (SNR) is defined by

$$\mathrm{SNR(dB)} = -20\log_{10}\sigma.$$

The WER of standard threshold detection is given by

$$\mathrm{WER} = 1 - \left(1 - Q(\frac{1}{2\sigma})\right)^n, \tag{6}$$

where the Q-function is defined as

$$Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty \exp(-\frac{u^2}{2})du.$$

Figure 1 shows the word error rate (WER) as a function of the signal-to-noise ratio (SNR) for three detection systems, a) offset-resistant threshold detection ($T=1$), b) offset/gain-resistant threshold detection ($T=2$), and c) analytic expression for a standard threshold detection method. The diagram shows that offset-resistant threshold detection degrades around 3 dB over the standard threshold detection system, and the offset/gain-resistant threshold detection loses around 1 dB. The performance of offset/gain-resistant threshold detection

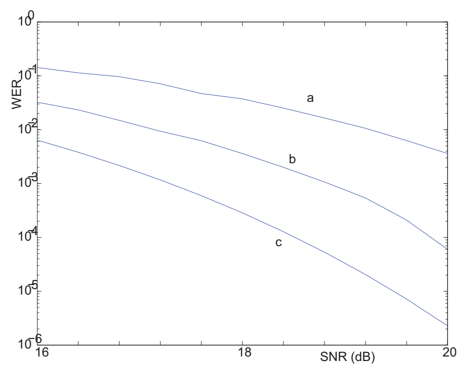

Fig. 1 *Word error rate (WER) as a function of the signal-to-noise ratio (SNR) for $q=2$ (binary) and $n=8$. The diagram shows the performance of a) offset-resistant threshold detection (T=1), b) offset/gain-resistant threshold detection (T=2), and c) standard threshold method (theory).*

($T=2$) significantly improves over the $T=1$ offset-resistant threshold detection case. A simple explanation is that, in the case $T=1$ the offset estimator, $\hat{b}$, is biased, which reduces the noise margin by around 3 dB. For $T=2$, both the gain and offset estimators are biased, but the bias of the gain estimator compensates for the offset estimator bias. More effort is needed to quantify these effects more precisely.

Conclusions: We have presented codes for q-ary channels with unknown gain, a, and offset, b. We have shown that codewords where T, $T>0$, reference symbols appear at least once, can be used for training the detection circuitry. We have enumerated the number of T-constrained q-ary n-length sequences. In the simplest case, $T=2$, each codeword has at least one symbol that represents the lowest (floor) signal value and at least one symbol that represents the highest (ceiling) signal value. Results of computer simulations have shown that the new coding and detection methods offer a performance that is completely resistant against offset and gain uncertainties with only a small loss in noise detection margin at the nominal values of offset and gain.

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